The Comprehensive Guide to Astrology Math

Analysis, explanation, and diagramming by Vera Gonzalez

This article is a work in progress. Inaccuracies may be present in both explanation and diagramming.

This article is targeted towards an audience that wants to deeply understand how real world Astronomy directly results in the divinatory information read by Astrologers around the globe. This is primarily presented in a mathematical and conceptual framework, but some components require pseudocode to be understood in their modern praxis.

As part of my creation of the IOS App Familiar Spirit, I created an Astronomy / Astrology math library from scratch in Swift. Doing so took a great deal of research, conceptualization, and code. The fruits of that labor are condensed for you here.

No prior knowledge of Astronomy is assumed, so this article is very verbose. I encourage you to digest it in parts, even taking your own notes.

Great inspiration in diagramming has been pulled from The Elements of House Division by Ralph William Holden, a masterful work of Astrology math that has been largely forgotten by contemporary Astrologers. I was only able to read it by visiting the Library of

Congress, but it clarified some of the higher complexity House Division mathematics. The Astrological community owes him a great debt for preserving this knowledge.

To see more of my work, or to contact me, visit vera.lgbt.

▼ Fundamental Building Blocks

This section will provide concepts that are essential to a complete understanding of all subsequent sections. Do not skip these concepts if you have any plans to implement Astrology Math in a codebase of your own.

▼ Angle Measurement

Let us define an AstroAngle as an angle that is represented internally in one of the following forms.

▼ degrees:

Familiar to anyone who has taken Geometry, degrees divide the circle into 360 even components. A degree is denoted using the $^{\circ}$ symbol. The result is a system that lends itself to highly intuitive angle math, an innovation that we can thank the Sumerians for.

▼ minutes:

A ‘minute’ is $\frac{1}{60}th$ of a degree. It is denoted using the $'$ symbol.

▼ seconds:

A ‘second’ is $\frac{1}{60}th$ of a minute. It is denoted using the $''$ symbol.

Degree based AstroAngles can be initialized in the form $\text{AstroAngle}(\text{degrees} : x)$ or the form $\text{AstroAngle}(\text{degrees} : x, \text{minutes} : y, \text{seconds} : z)$.

▼ hours:

Unlike degrees, hours divide the circle into 24 even components. An hour is denoted using the symbol h. The result is that $1\text{h} = 15^{\circ}$. While less common than degrees, it is still used widely in Astronomy by convention.

▼ minutes:

A ‘minute’ is $\frac{1}{60}th$ of an hour. It is denoted using the m symbol.

▼ seconds:

A ‘second’ is $\frac{1}{60}th$ of a minute. It is denoted using the s symbol.

Hour based AstroAngles can be initialized in the form $\text{AstroAngle}(\text{hours} : x)$ or the form $\text{AstroAngle}(\text{hours} : x, \text{minutes} : y, \text{seconds} : z)$.

Note that traditionally, we do not employ radians in our computations. If you are attempting to implement this math in software, that might change, but fundamentally you must also be able to work with these value forms.

Let us also define a method bound which will modify a given value to keep it within the expected range of an AstroAngle. While extremely simple, this method is also extremely useful in performing Astronomy math.

▼ $\text{bound}(\theta)$:

$\text{while } \theta < 360^o :$

$\theta = \theta + 360^o$

$\text{while } \theta > 360^o :$

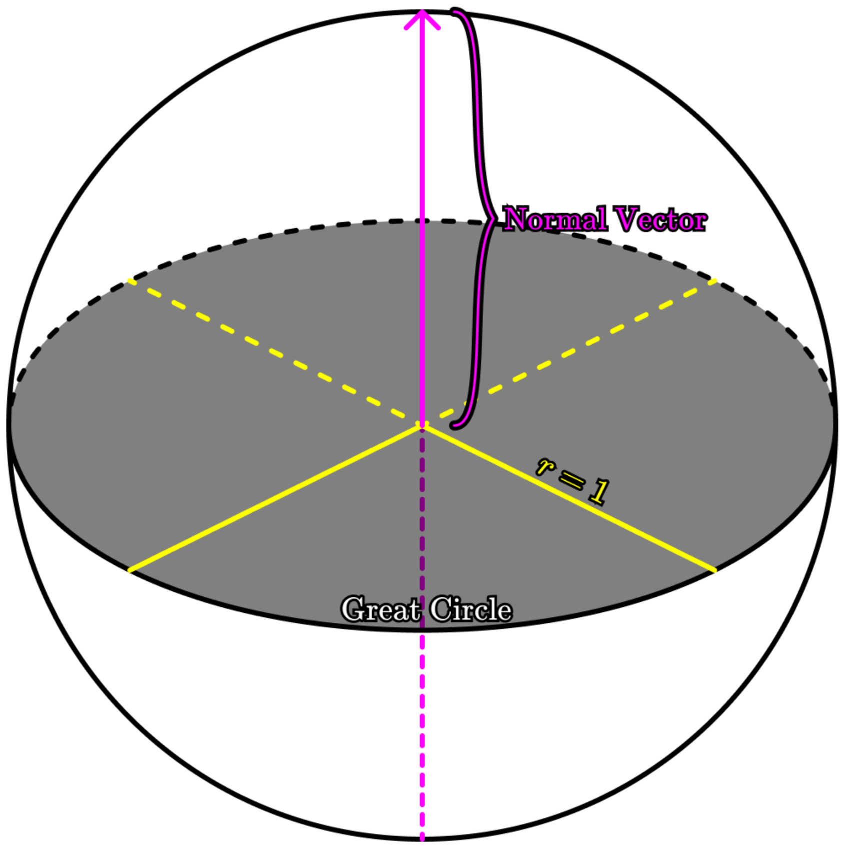
$\theta = \theta - 360^o$

$\text{return } \theta$

▼ Great Circles & Normal Vectors

There is a great deal of spherical trigonometry in the matter of Astrology math. Two concepts fundamental to an understanding of that trigonometry are the notions of Great Circles and Normal Vectors. Great Circles are any circles that, when drawn on a sphere, bisect that sphere into two equal halves. Normal Vectors have a more nuanced meaning in other fields such as computer graphics and game design, but for our purpose they can be thought of simply as Unit Vectors which are perpendicular to a given Great Circle. The reason we care deeply about these concepts is because our Coordinate Systems primarily employ Great Circles as their reference points. Additionally, by taking the Cross Product of any two Normal Vectors, we can find the intersections of the Great

Circles they correspond to. This is an essential methodology in the matter of House Division.



▼ Ecliptic Plane & Zodiacal Signs

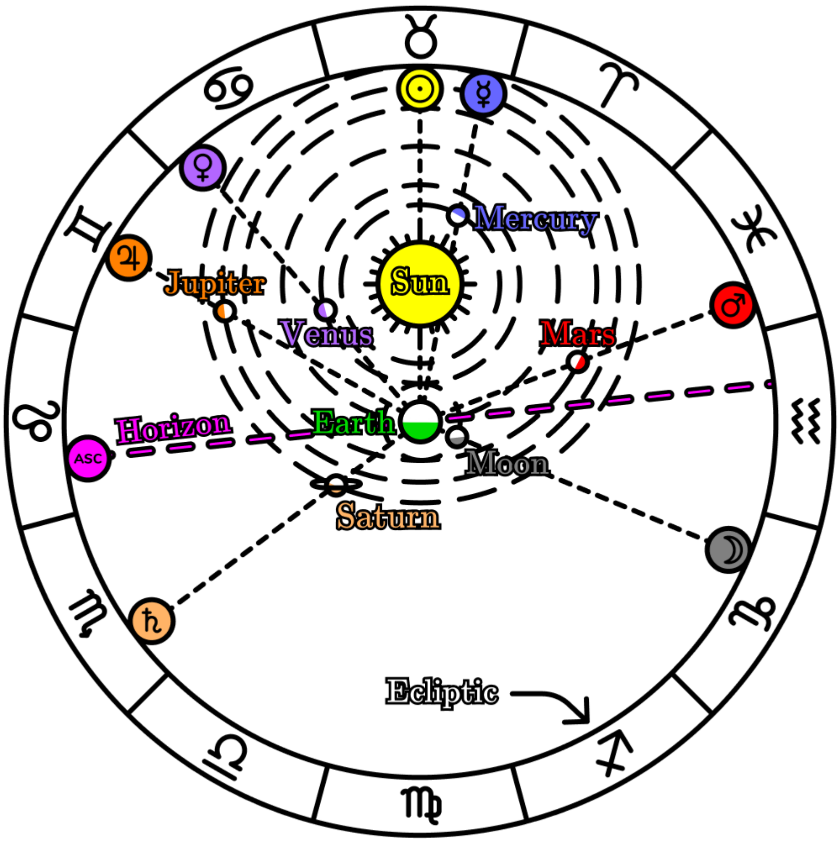
Recall for a moment that in diagrams of the Solar System we see that although each planet exists in a different orbit, they all roughly share a plane. If we imagine this plane as our 'ground' it can be easily seen that no planet moves drastically "above" or "below" another.

The Zodiacal Signs lie *on* the Ecliptic Plane, and simply divides it as a circle into twelve equal sections of 30^{o} each.

The locations of planets seen plotted on a traditional horoscope do not showcase planetary location through Heliocentric orbit, but instead apparent location from a Geocentric perspective.

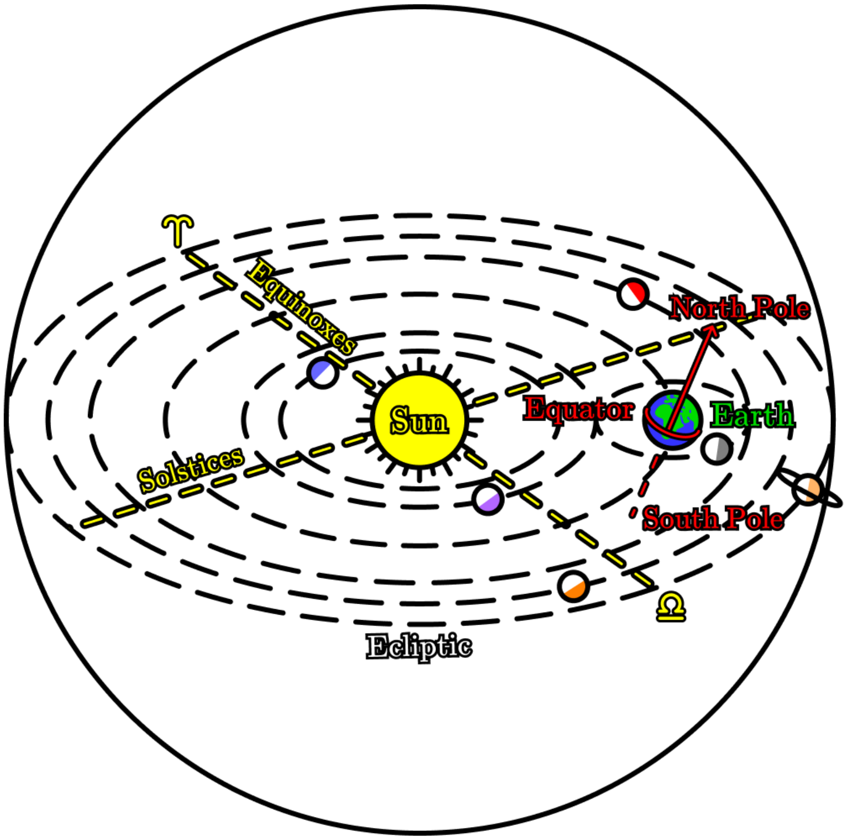
Here, I illustrate an example chart and the solar system in the same image such that the relationship between the the Ecliptic Plane and the Astrological Chart can be understood.

Let me also note that traditionally, the Horizon is made to appear completely horizontal on a Chart, but this convention is being broken here to showcase that the angle of the Horizon does not need to match the angle with which the Earth faces the Sun.



▼ Solstices & Equinoxes

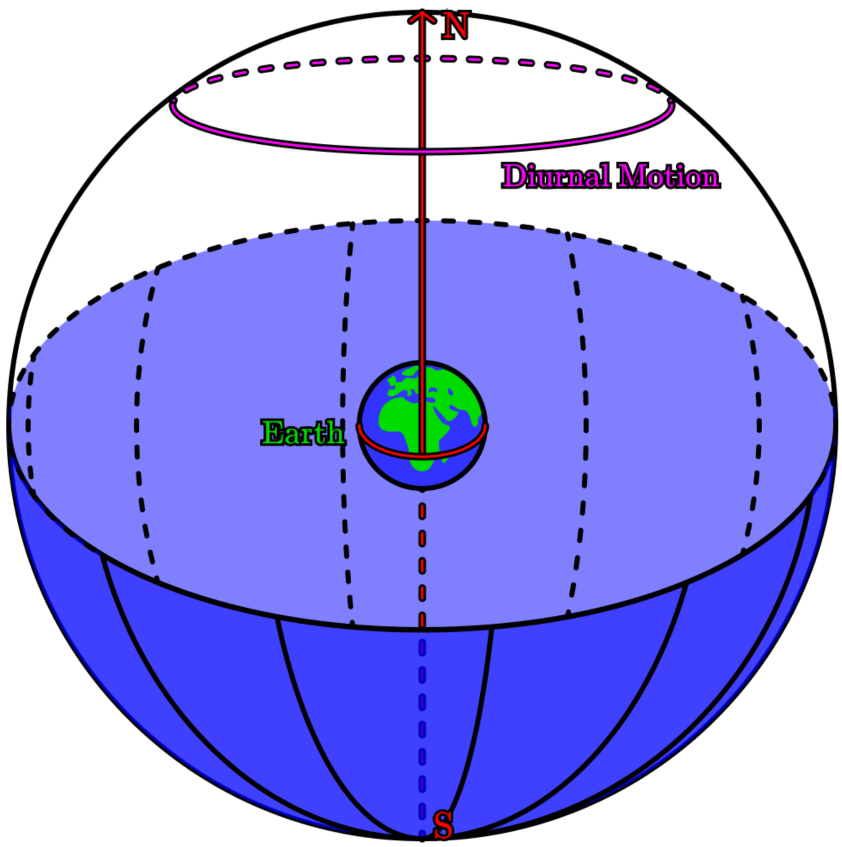
Essential to an understanding our Coordinate Systems is an understanding of the Equinoxes. Put simply, Solar Equinoxes occur when the apparent position of the Sun is directly above the Earth's Equator. In other words, the moments in time where the Earth's Equator is pointing directly towards the Sun. This happens twice a year. By contrast, the Solstices occur when Earth's Equator is pointing as far from the sun as is possible. This also happens twice a year.



From this diagram it can be gleamed that the tilt of the Earth's Axis is the cause of this behavior. Because this behavior directly dictates Earth's seasons, it should be no surprise that these locations on the Ecliptic Plane became essential reference points when discussing Astronomical phenomena, even in antiquity.

▼ The Celestial Sphere & Diurnal Motion

The Celestial Sphere represent a way on conceptualizing the locations of heavenly bodies in the night sky. Under this conception, the Earth is enclosed by a much larger, concentric sphere. On this sphere, we see stars, the Moon, the Sun, and all other heavenly bodies. This is the intuitive and original understanding of the night sky. Diurnal Motion refers to the apparent movement of stars and other bodies through this sphere as a product of the Earth's axial rotation.



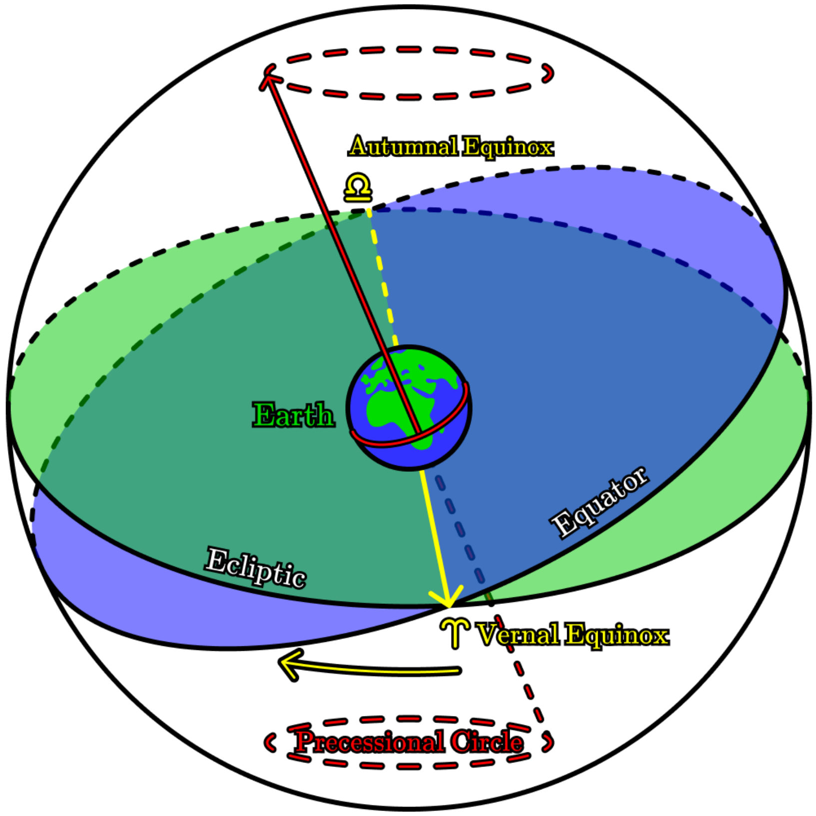
By performing a long exposure photograph focused on Polaris, the North Star, we can see that all other stars trace concentric circles around the Earth's axis as it rotates, regardless of the location of the Observer. This is Diurnal Motion.

▼ Precession

When Solstices & Equinoxes were first employed as reference points, the phenomenon of Precession was not known. It was assumed implicitly that the locations of the Solstices & Equinoxes on the Ecliptic Plane were unchanging, and these became the locations upon which all other Astrological data was projected. The Vernal Equinox was dubbed “Aries 0°” while the Autumnal Equinox was dubbed “Libra 0°”, named by the constellations they were home to.

Precession describes the movement of the Earth’s axis through the Precessional Circle, which can be conceptually likened to the wobbling of a top. This wobble is a slow one, completing a full circle once every 25,788 years. It should be no surprise, then, that the discrepancy was not noticed for some time. In the meanwhile, these reference points and the associated constellation names had already cemented themselves in Astrological tradition. This is the discrepancy between “Tropical” and “Sidereal” Astrology. Tropical Astrology employs the original reference point, the Vernal Equinox, while Sidereal Astrology focuses on the point in the night sky where the Vernal Equinox once was as its reference point: The Aries Constellation.

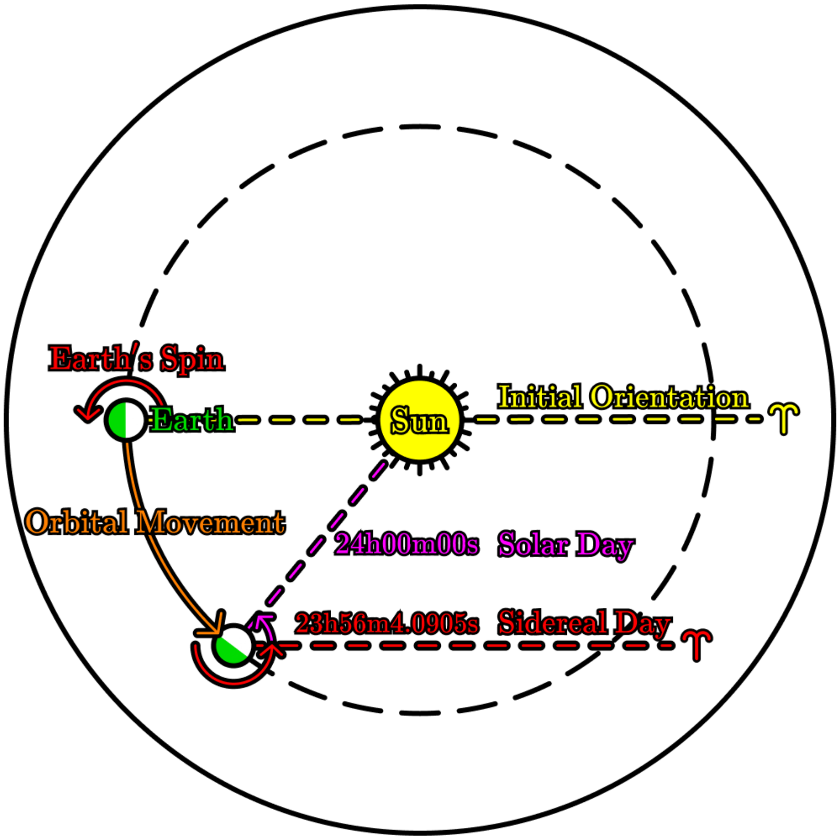
Additionally, note that as a result of this movement, the so called North Star changes over time. Right now, our North Star is Polaris, but will become Deneb in about 8,000 years. Eventually, it will become Polaris again.



▼ Sidereal Days & Solar Days

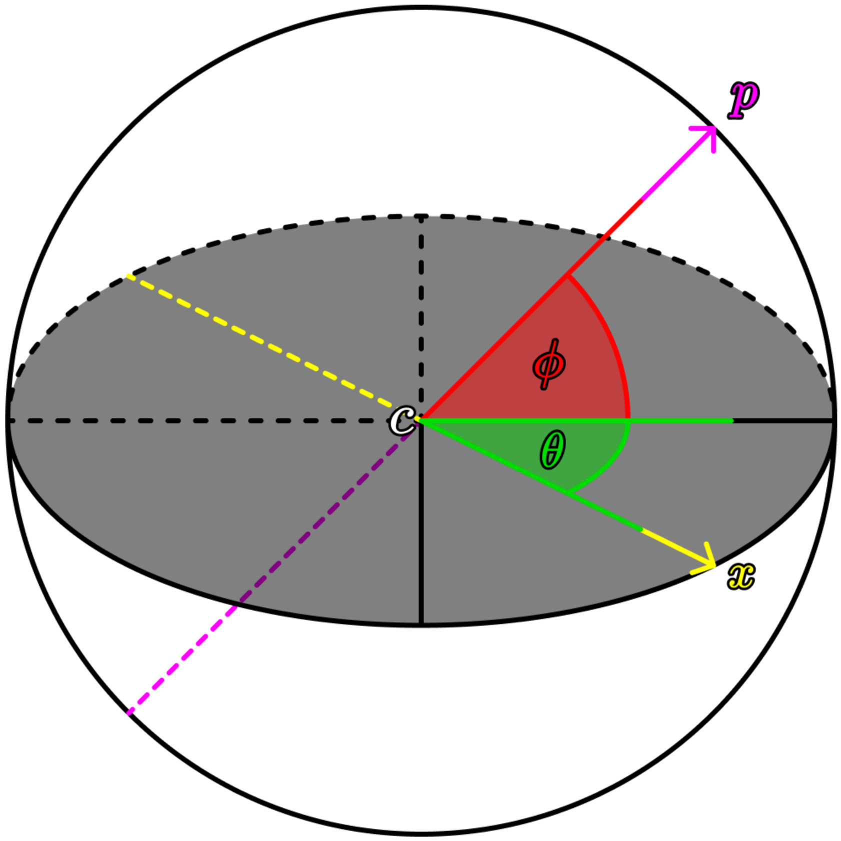
Sidereal Time is an important concept to understand in the context of Astronomical observation. When we typically think of a “Day”, we mean a Solar Day. This is the time it takes for the Sun to move from directly overhead a full 360^{o} back to directly overhead. By contrast is the Sidereal Day, which describes the duration for a fixed star (far further away from us that our Sun) to reach the same point in the sky. The discrepancy between these two values is a direct product of our Orbital Movement around the Sun. Our angle both to our Sun and distant stars changes as a result of this movement, but because the distant stars are any number of light years away, the change in their apparent angle is essentially zero.

When describing Astronomical observations of fixed stars, Observers often employ the use of Local Sidereal Time rather than of Greenwich Mean Time because it enables a single “Day” to correspond to an exact, full rotation. For this reason, you will often see Sidereal Time tables in ephemerides.



Coordinate Systems

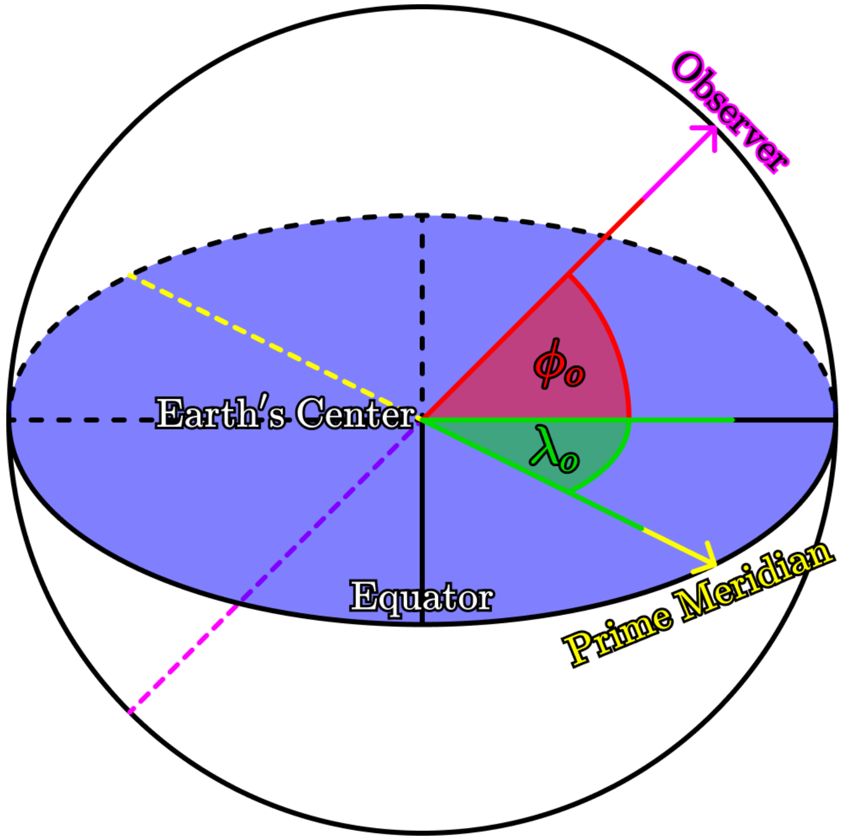
Coordinate Systems are the way we make sense of locations on the Celestial Sphere. Let us define a Coordinate as a combination of two AstroAngles, θ and ϕ. Together, the two angles can represent any point or unit vector on a sphere, p where $|p| = 1$. From the chosen reference point c and starting vector x, we use θ to represent the crosswise angle and ϕ to represent the vertical angle. Note that for the purposes of our computations, we imply that $-180^o \leq \theta \leq 180^o$ and $-90^o \leq \phi \leq 90^o$. The reasoning for this will become more clear when we begin the challenge of Coordinate Conversion, but is primarily for the sake of maintaining consistent quadrants during trigonometry. We do not need to denote the length of the vector p because non-unit vectors are out of scope.



As a disclaimer, it should be noted the conventions that I employ here are not universal in Astronomy. Different sources employ a wide array of starting vectors and measure both clockwise and anti-clockwise, allowing different ranges in their values between various systems. Here, all positive values of θ are clockwise and all positive values of ϕ are 'above' the plane on which x sits unless specified otherwise. This is the convention for Geographic Coordinates and so should be more intuitive.

▼ Geographic Coordinates

Geographic Coordinates are the type that we see on Google Maps, the ones we use to navigate across the globe and which mark locations on it.



c : The center of the Earth

x : The Longitude of Greenwich, England (Prime Meridian)

θ : Longitude (λ_o)

Typically represented in the form $\mathrm{XX^oYY'ZZ''}$.

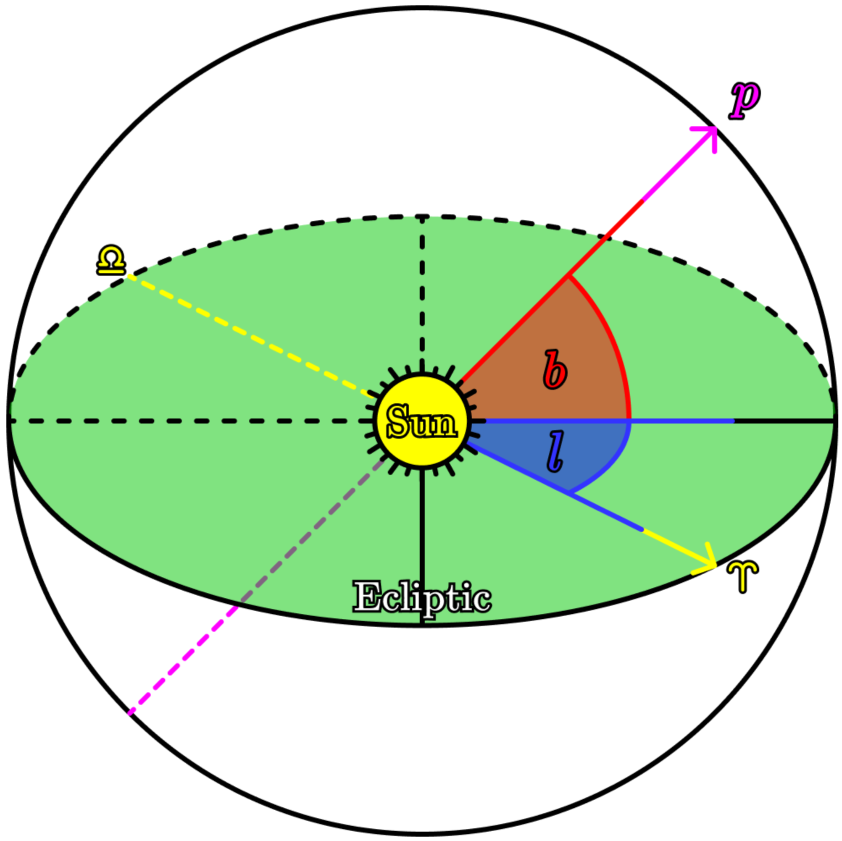
ϕ : Latitude (ϕ_o)

Typically represented in the form $\mathrm{XX^oYY'ZZ''}$.

p : Location on Earth

▼ Heliocentric Ecliptic Coordinates

Heliocentric Ecliptic Coordinates define where a celestial body is in relation to the Ecliptic Plane from the perspective of the Sun.



c : The Sun

x : Vernal Equinox

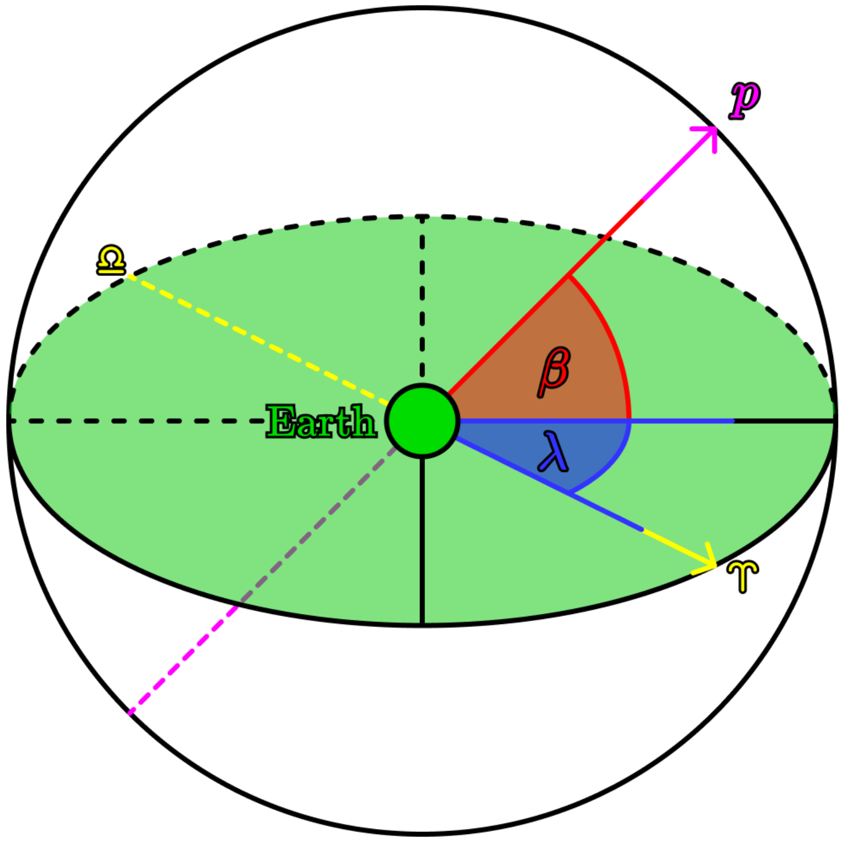
θ : Longitude (l)

ϕ : Latitude (b)

p : Location of celestial object

▼ Geocentric Ecliptic Coordinates

Geocentric Ecliptic Coordinates define where a celestial body is in relation to the Ecliptic Plane from the perspective of the Earth. This is the view ancient astronomers were most interested in, as the planets (wanderers) in the night sky all through this great circle with low magnitude latitudes.



c : The Earth

x : Vernal Equinox

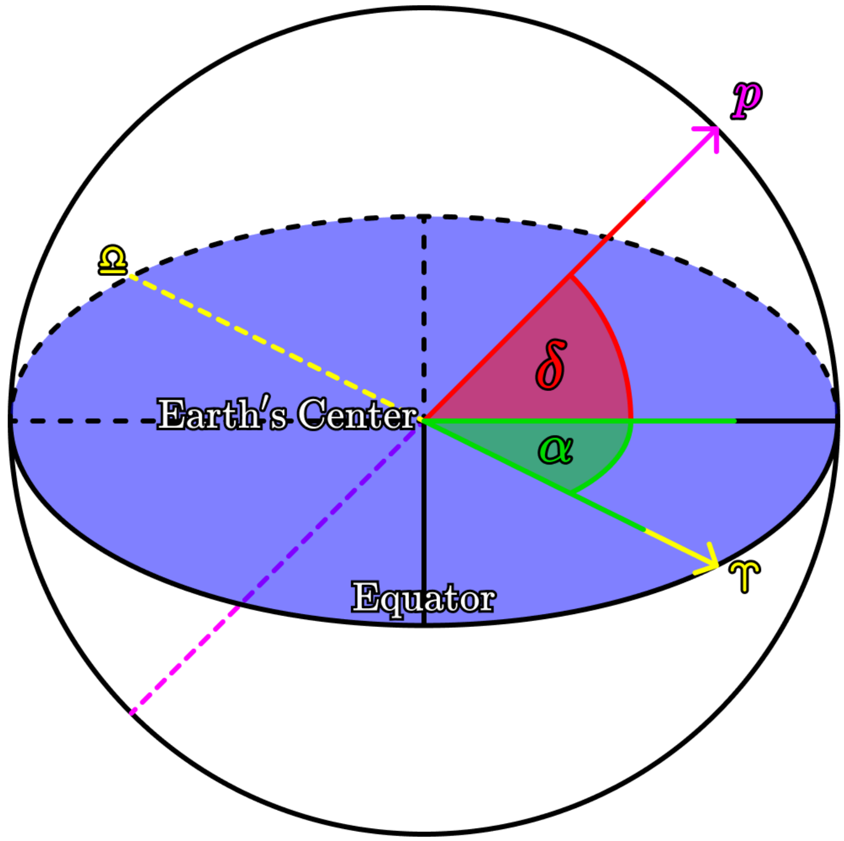
θ : Longitude (λ)

ϕ : Latitude (β)

p : Location of celestial object

▼ Equatorial Coordinates

Equatorial Coordinates measure the location of celestial bodies using the Equator as the primary reference point. The starting point x along the Equator requires an understanding of Geocentric Ecliptic Coordinates to be understood.



c : The center of the Earth

x : Vernal Equinox

θ : Right Ascension (α)

Typically represented in the form $\mathrm{XXhYYmZZs}$.

ϕ : Declination (δ)

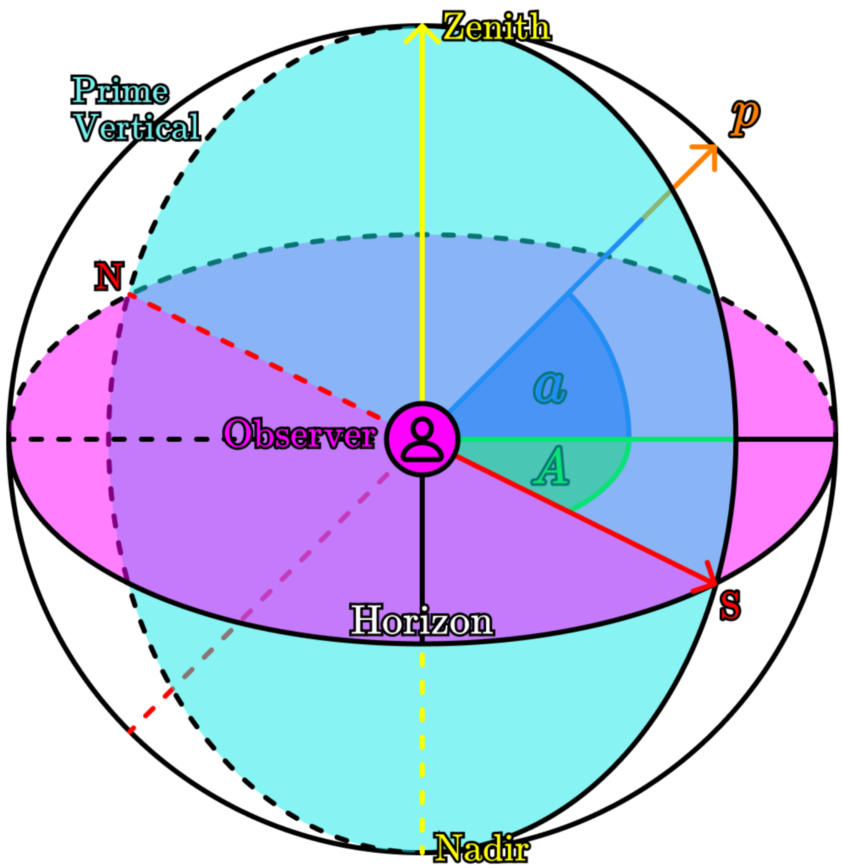
Typically represented in the form $\mathrm{XX^oYY'ZZ''}$.

p : Location of celestial body

Horizontal Coordinates

Horizontal Coordinates define where something in the sky is from an Observer's point of view. Note that because on a celestial scale, the diameter of the Earth is minuscule, positioning an Observer in the center of the Earth and imagining that the Earth is transparent is sufficient here. Correcting for this distance is out of scope and does not prevent us from achieving high accuracy results. Note that while the Observer may be in the center of the Earth, that center can be 'drilled to' from *any* Geographic Coordinate and need not be axis aligned in any way with the Equator, Ecliptic, etc.

Also note that as a result of this definition, any celestial object with a negative Horizontal Altitude ϕ is considered below the horizon of the Observer, and by extension unable to be seen.



c : The Observer themselves

x : True South for the given Observer

θ : Azimuth (A)

Typically represented in the form $\text{XX}^{\text{o}}\text{YY}'\text{ZZ}''$.

Measured anti-clockwise.

ϕ : Altitude (a)

Typically represented in the form $\text{XX}^{\text{o}}\text{YY}'\text{ZZ}''$.

p : Location in the night sky

Note that in this Coordinate System, the Normal Vector and its diametric opposite are both given a special name, the Zenith and the Nadir.

▼ Observers & Time

Let us simply define an Observer as a combination of a Geographic Coordinate in the form (λ_o, ϕ_o) and a local date and time (timestamp). This is the basis for the construction of an Astrological Chart, but is equally the basis for many of the properties that we will use in that construction. Those properties are detailed here. In addition to forming the basis of these computed properties, Observers and their properties will also be used in Coordinate Conversion and the computation of Astrological Houses.

Note that this section is dense mathematically. Even the most renowned modern Astrological texts gloss over these computations in particular and instead rely on Ephemerides.

▼ Greenwich Mean Time (GMT) $(\mathcal{G})$

This is the singular component of computation wherein I would recommend the employment of a third party library. For our purposes, GMT is close enough to UTC that they can be considered interchangeable. Despite this, determining GMT or UTC can be an enormous task. This is not due to any particular difficulty in mathematics (Observer longitude need only be accounted for), but due to the sheer complexity in human time-keeping systems. Some countries still to this day do not employ Gregorian time, other switched at different periods in history, others have jumped across time zones and date lines. A more thorough explanation with more detailed examples can be found here, but let it suffice to say that a third party extension should be employed. I used SwiftDate in my own implementation.

For our purposes and notation, we will agree that the individual components of a GMT date or timestamp can be represented using subscripts as follows:

- $\mathcal{G}_Y$ for Year
- $\mathcal{G}_M$ for Month
- $\mathcal{G}_D$ for Day

- $\mathcal{G}_h$ for Hour
- $\mathcal{G}_m$ for Minute
- $\mathcal{G}_s$ for Second

Further precision beyond this point is not required for our purposes.

Julian Day (J)

While the Gregorian Calendar is by far the most pervasive of human time systems today, it is not the most useful in all cases. In Astronomy, there is often the desire to represent the current time as a single number, representing the time since a given day. The Julian Day is, in this way, the primordial Unix Time. Note that this equation is pulled directly from SwiftDate and is intended to be used with UTC Time, but again I emphasize that for our purposes UTC and GMT are indistinguishable.

$\text{Compute\_Julian}(\mathcal{G}):$

1. $h = \mathcal{G}_h + \mathcal{G}_m/60 + \mathcal{G}_s/3600$
2. $J = 367 \times \mathcal{G}_Y - \text{floor}(7 \times (\mathcal{G}_Y + \text{floor}((\mathcal{G}_M + 9)/12))/4)$
3. $J = J - \text{floor}(3 \times (\text{floor}((\mathcal{G}_Y + (\mathcal{G}_M - 9)/7)/100) + 1)/4)$
4. $J = J + \text{floor}(275 \times \mathcal{G}_M/9) + \mathcal{G}_D + 1721028.5 + h/24$
5. $\text{return } J$

Greenwich Mean Sidereal Time (GMST) (θ_G)

To understand the desire for GMST, it is important to understand the difference between Solar Time and Sidereal Time. Once Sidereal Time is understood, it should also become apparent that its computation should prove strange and troublesome for any party not directly there to measure it. Therefore we construct a simplified methodology for determining the Sidereal Time in Greenwich, which is to say along the Prime Meridian. Later, θ_G can be adjusted for location to find θ_L. Note that as input to this computation we require a precise Julian Day for a given GMT.

$\text{Compute\_GMST(J)}:$

1. $y = (J - 2451545)/36525$

   This computes the exact number of years since January 1st, 2000, Noon. Note that Julian Days are not necessarily required to perform this computation but are an easy way to arrive at it.

2. $\mathrm{sd} = ((-2.454 \times 10^{-9}) \times (y - 7507.52) \times (y^2 + 7914.67y + 22.086) \times (y^2 + 406.653y + (5.92541 \times 10^7)) + (J \mod 1.0 + 0.5) \times 86400)/240d = ((-2.454 \times 10^{-9}) \times (y - 7507.52) \times (y^2 + 7914.67y + 22.086) \times (y^2 + 406.653y + (5.92541 \times 10^7)) + (J \mod 1.0 + 0.5) \times 86400)/240$

3. $\theta_G = \mathrm{bound}(\mathrm{sd})$

4. return θ_G

Local Mean Sidereal Time (LMST) (θ_L)

Often simply represented as LST, the LMST (θ_L) of an Observer is simply the GMST of the Observer's GMT, adjusted for Observer Longitude (λ_o). With this in mind

$\mathrm{Compute\_LMST}(\theta_G, \lambda_o)$:

1. $\theta_L = \mathrm{bound}(\theta_G + \lambda_o)$

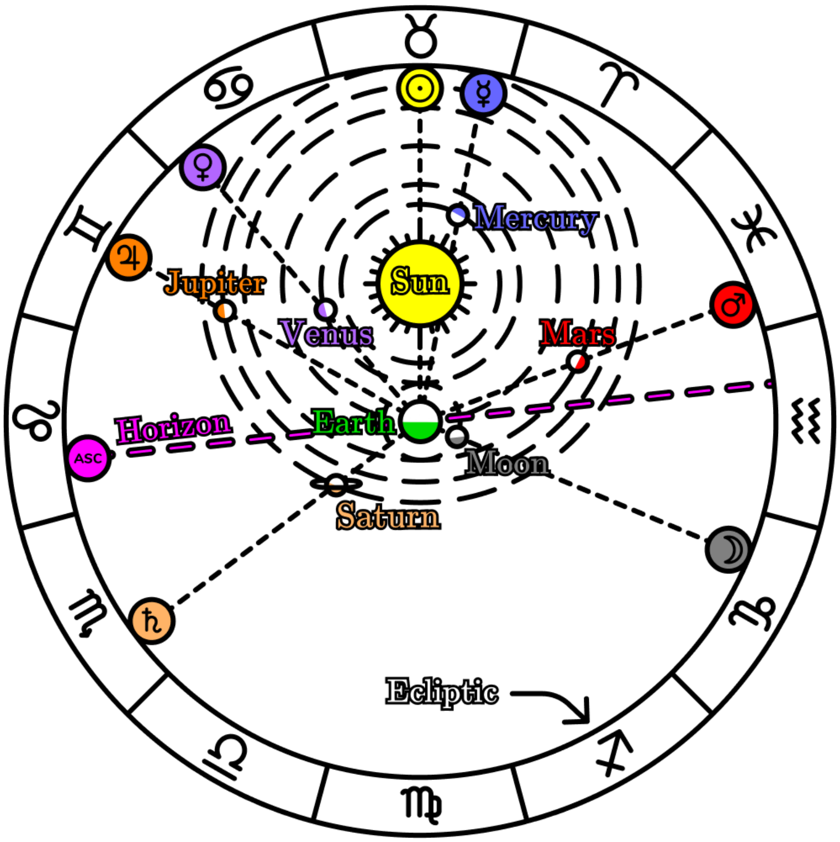
2. return θ_L

Coordinate Conversion

Let us define that for the following conversion procedures, we have been provided an Observer which bears its specified properties.

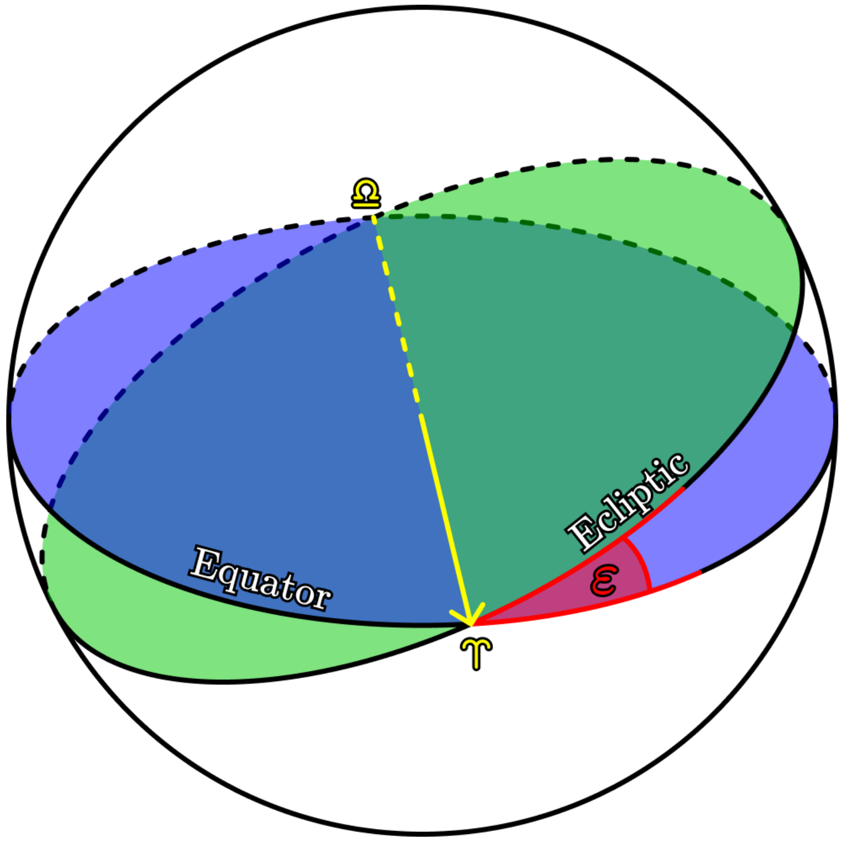
Heliocentric Ecliptic $\rightarrow$ Geocentric Ecliptic

Recall for a moment the first diagram present in this article, the projection of the Ecliptic Plane. While this diagram shows us what the Ecliptic Plane means in the context of Astrology, it also directly demonstrates the means for representing a Heliocentric Ecliptic Coordinate in Geocentric Ecliptic Space. By simply obtaining the Heliocentric Coordinates for each Planet and representing them in Rectangular form, the reference point can be shifted through simple addition to that of the Earth, where we can then re-infer the Rectangular positions as Angular Coordinates.



▼ Geocentric Ecliptic ↔ Equatorial

Let us define the following procedures for Geocentric Ecliptic Coordinates in the form $(\theta, \phi) = (\gamma, \beta)$ and Equatorial Coordinates in the form $(\theta, \phi) = (\alpha, \delta)$. Because in essence, these two Coordinate Systems share the same starting vector x, only the reference point needs to be adjusted for. This starting vector is the Vernal Equinox, which we denote with the Aries symbol ♈. The result is that we simply rotate a given coordinate around this central axis.



▼ $\text{GeocentricEcliptic\_to\_Equatorial}_{\text{Observer}}(\gamma, \beta)$:

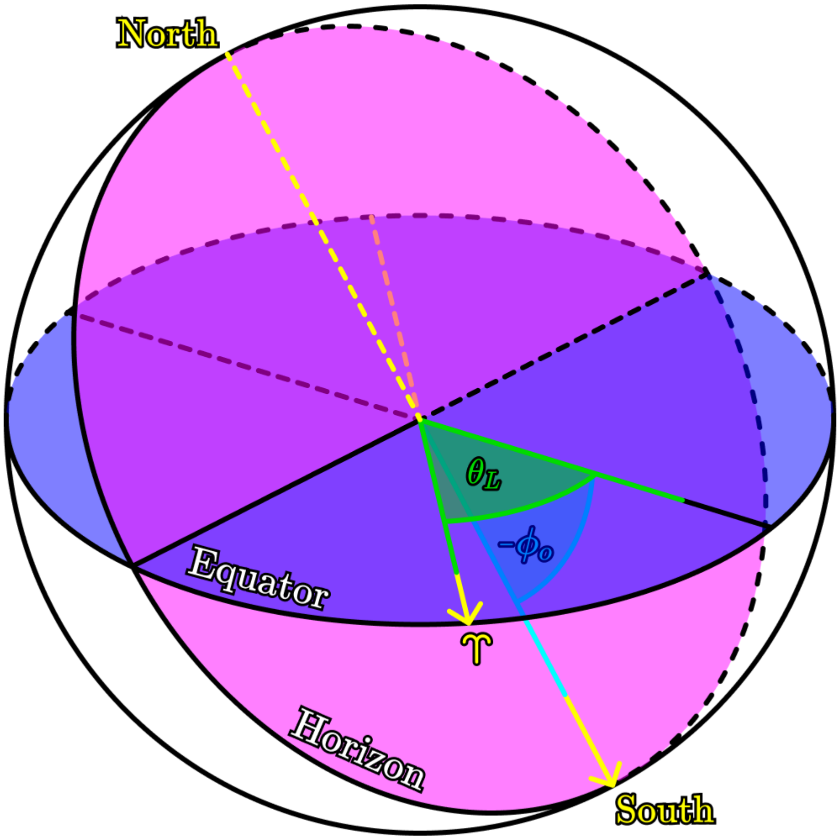
1. $\delta = \arcsin(\sin(\beta) \times \cos(\varepsilon) + \cos(\beta) \times \sin(\varepsilon) \times \sin(\gamma))$
2. $\alpha = \text{arctan}_2(\sin(\gamma) \times \cos(\varepsilon) - \tan(\beta) \times \sin(\varepsilon), \cos(\gamma))$
3. $\text{return } (\alpha, \delta)$

▼ $\text{Equatorial\_to\_GeocentricEcliptic}_{\text{Observer}}(\alpha, \delta)$:

1. $\beta = \arcsin(\sin(\delta) \times \cos(\varepsilon) - \cos(\delta) \times \sin(\varepsilon) \times \sin(\alpha))$
2. $\gamma = \text{arctan}_2(\sin(\alpha) \times \cos(\varepsilon) + \tan(\delta) \times \sin(\varepsilon), \cos(\alpha))$
3. $\text{return } (\gamma, \beta)$

▼ Equatorial ↔ Horizontal

Let us define the following procedures for Equatorial Coordinates in the form $(\theta, \phi) = (\alpha, \delta)$ and Horizontal Coordinates in the form $(\theta, \phi) = (A, a)$. Unlike our previous conversion challenge, these conversions can differ across two axes simultaneously, making their transformation more complex.



▼ $\text{Equatorial\_to\_Horizontal}_{\text{Observer}}(\alpha, \delta)$:

1. $h = \theta_L - \alpha$

   This computes the Local Hour Angle of the celestial body in question.

2. $a = \arcsin(\sin(\delta) \times \sin(\phi_o) + \cos(\delta) \times \cos(\phi_o) \times \cos(h))$
3. $A = \arctan_2(-\sin(h) \times \cos(\delta) \ / \ \cos(a) \ , \ [\sin(\delta) - \sin(\phi_o) \times \sin(a)] \ / \ [\cos(\phi_o) \times \cos(a)])$
4. $\text{return } (\text{A}, \text{a})$

▼ $\text{Horizontal\_to\_Equatorial}_{\text{Observer}}(\text{A}, \text{a})$

1. $\delta = \arcsin(\sin(a) * \sin(\phi_o) + \cos(a) * \cos(\phi_o) * \cos(A))$

2. $h = \arctan_2(-\sin(A) * \cos(a) / \cos(\delta), (\sin(a) - \sin(\delta) * \sin(\phi_o)) / \cos(\delta) * \cos(\phi_o))$

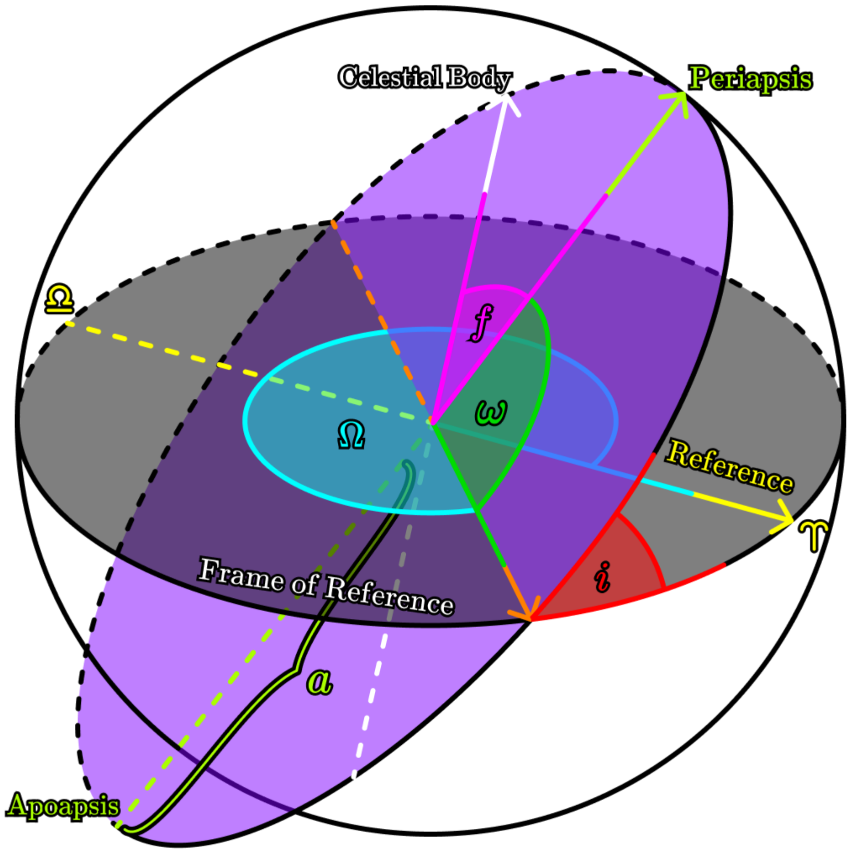
   This computes the Local Hour Angle of the celestial body in question.

3. $\alpha = \theta_L - h$
4. $\text{return } (\delta, \alpha)$

Orbital Elements

The Orbital Elements of a celestial body determine the shape and orientation of its orbit. The only reason we require an understanding of them is so that we can use them in the computation of Planetary Placements, where they are referenced by referring to a given Planet as the variable p. Beyond this scope, they serve very little utility to us as Astrologers. For that reason, this section is conceptually brief and mathematically verbose.

Note that the Orbit defined by the Elements in question is not a Great Circle. Additionally it will be seen that because most Planets lie within the Ecliptic Plane to an extent, computed Inclination values are generally low. This is true except in the case of the Moon, which experiences a greater range in Inclination.



Ω : Longitude of Ascending Node

i : Inclination

ω : Argument of Periapsis

a : Semi-major axis

e : Eccentricity

M : Mean anomaly

f : True anomaly for the given epoch t_0

r : The distance of the body from the center of our system

Prior to any computations, we must also define a modified Julian Day, one which creates a value more familiar.

▼ $\text{Compute\_MJD}(J)$

$d = J - 2451543.5$

return d

Assume now that Compute_MJD has been run for a given Observer's Julian Day and the resulting value d is stored locally.

Note that for the values $\Omega,\ i,\ \omega$, and M, we compute their values here in raw degrees which can subsequently be fed to an AstroAngle object instance. Note also that we implicitly utilize the bound function in obtaining our result. Because the other values are not AstroAngles, the bound function is not needed.

▼ Sun_Orbitals

$\Omega : 0$

$i : 0$

$\omega : 282.9404 + (4.70935 \times 10^{-5}) \times d$

$a : 1$

$e : 0.016709 - (1.151 \times 10^{-9}) \times d$

$M : 356.0470 + 0.9856002585 \times d$

▼ Moon_Orbitals

$\Omega : 125.1228 - 0.0529538083 * d$

$i : 5.1454$

$\omega : 318.0634 + 0.1643573223 \times d$

$a : 60.2666$

$e : 0.0549$

$M : 115.3654 + 13.0649929509 \times d$

▼ Mercury_Orbitals

$\Omega : 48.3313 + (3.24587 \times 10^{-5}) \times d$

$i : 7.0047 + (5.00 \times 10^{-8}) \times d$

$\omega : 229.1241 + (1.01444 \times 10^{-5}) \times d$

$a : 0.387098$

$e : 0.205635 + (5.59 \times 10^{-10}) \times d$

$M : 168.6562 + 4.0923344368 \times d$

▼ Venus_Orbitals

$\Omega : 76.6799 + (2.46590 \times 10^{-5}) \times d$

$i : 3.3946 + (2.75 \times 10^{-8}) * d$

$\omega : 54.8910 + (1.38374 \times 10^{-5}) \times d$

$a : 0.723330$

$e : 0.006773 - (1.302 \times 10^{-9}) \times d$

$M : 48.0052 + 1.6021302244 \times d$

▼ Mars_Orbitals

$\Omega : 49.5574 + (2.11081 \times 10^{-5}) \times d$

$i : 1.8497 - (1.78 \times 10^{-8}) \times d$

$\omega : 286.5016 + (2.92961 \times 10^{-5}) \times d$

$a : 1.523688$

$e : 0.093405 + (2.516 \times 10^{-9}) \times d$

$M : 18.6021 + 0.5240207766 \times d$

▼ Jupiter_Orbitals

$\Omega : 100.4542 + (2.76854 \times 10^{-5}) \times d$

$i : 1.3030 - 1.557 \times 10^{-7} \times d$

$\omega : 273.8777 + 1.64505 \times 10^{-5} \times d$

$a : 5.20256$

$e : 0.048498 + 4.469 \times -9 \times d$

$M : 19.8950 + 0.0830853001 \times d$

▼ Saturn_Orbitals

$\Omega : 113.6634 + 2.38980 \times 10^{-5} \times d$

$i : 2.4886 - 1.081 \times 10^{-7} \times d$

$\omega : 339.3939 + (2.97661 \times 10^{-5}) \times d$

$a : 9.55475$

$e : 0.055546 - (9.499 \times 10^{-9}) \times d$

$M : 316.9670 + 0.0334442282 \times d$

▼ Uranus_Orbitals

$\Omega: 74.0005 + (1.3978 \times 10^{-5}) \times d$

$i: 0.7733 + (1.9 \times 10^{-8}) \times d$

$\omega: 96.6612 + (3.0565 \times 10^{-5}) \times d$

$a: 19.18171 - (1.55 \times 10^{-8}) \times d$

$e: 0.047318 + (7.45 \times 10^{-9}) \times d$

$M: 142.5905 + 0.011725806 \times d$

▼ Neptune_Orbitals

$\Omega: 131.7806 + (3.0173 \times 10^{-5}) \times d$

$i: 1.7700 - (2.55 \times 10^{-7}) \times d$

$\omega: 272.8461 - (6.027 \times 10^{-6}) \times d$

$a: 30.05826 + (3.313 \times 10^{-8}) \times d$

$e: 0.008606 + (2.15 \times 10^{-9}) \times d$

$M: 260.2471 + 0.005995147 \times d$

There are additionally a few higher order values that must also be computed for each Planet before positions can be determined.

▼ Compute_E

$E_0 = M + \text{AstroAngle}(\text{radians}: e \times \sin(M) \times (1 + e \times \cos(M)))$

$\text{if Sun}:$

$\quad \text{return } E_0$

$\text{else}:$

$\quad E_1 = 0^{\circ}$

$\quad \text{while } E_0 > 0.05^{\circ} \text{ and } \text{abs}(E_0 - E_1) > 0.001^{\circ}$

$\quad\quad E_1 = E_0 - (E_0 - \text{AstroAngle}(\text{radians}: e \times \sin(E_0)) - M)/(1 - e \times \cos(E_0))$

$\quad\quad E_0 = E_1$

$\quad \text{return } E_0$

- Compute_f_r()

$$x_p = a \times \cos(E) - e$$

$$y_p = a \times \sqrt{1 - e^2} \times \sin(E)$$

$$f = \text{atan2}(y_p, x_p)$$

$$r = \sqrt{x_p^2 + y_p^2}$$

$$\text{return } f, r$$

~~Planetary Placements~~

Once the Orbital Elements for each Planet are computed for a given Observer, we must subsequently infer Heliocentric and Geocentric Coordinates for each Planet, so that they can be placed on the Astrological Chart.

Heliocentric Placements

Here, we utilize the Orbital Elements of a given Planet (p) to place it on the Heliocentric Ecliptic Plane.

- Compute_Heliocentric(p)

$$x_h = r \times (\cos(N) \times \cos(f + w) - \sin(N) \times \sin(f + w) \times \cos(i))$$

$$y_h = r \times (\sin(N) \times \cos(f + w) + \cos(N) \times \sin(f + w) \times \cos(i))$$

$$z_h = r \times (\sin(f + w) \times \sin(i))$$

$$\text{latitude}_h = \text{AstroAngle}(\text{radians} : \text{atan2}(z_h, \sqrt{x_h^2 + y_h^2}))$$

$$\text{longitude}_h = \text{AstroAngle}(\text{radians} : \text{atan2}(y_h, x_h))$$

$$\text{return } (\text{latitude}_h, \text{longitude}_h)$$

Perturbations

The Planetary Orbits do not exist in isolation from one another. While we can define them in isolation to an extent, there are extremely massive Planets in our solar system which have gravitational effects on one another. Therefore before we're able to obtain a Placement from a Geocentric perspective, we must take these Perturbations into account.

▼ Moon

Before computing the Perturbations for the Moon, we recognize the following shorthands:

L_s = Sun's Mean Longitude

L_m = Moon's Mean Longitude

M_s = Sun's Mean Anomaly

M_m = Moon's Mean Anomaly

$D = L_m - L_s$ = Moon's Mean Elongation

$F = L_m - N_m$ = Moon's Argument of Latitude

To account for the Moon's Perturbations, we first add the following terms to its Heliocentric Longitude

$$-1.274^o \times \sin(M_m - 2 \times D)$$

$$+0.658^o \times \sin(2 \times D)$$

$$-0.186^o \times \sin(M_s)$$

$$-0.059^o \times \sin(2 \times M_m - 2 \times D)$$

$$-0.057^o \times \sin(M_m - 2 \times D + M_s)$$

$$+0.053^o \times \sin(M_m + 2 \times D)$$

$$+0.046^o \times \sin(2 * D - M_s)$$

$$+0.041^o \times sin(M_m - M_s)$$

$$-0.035^o \times \sin(D)$$

$$-0.031^o \times \sin(M_m + M_s)$$

$$-0.015^o \times \sin(2 \times F - 2 \times D)$$

$$+0.011^o \times \sin(M_m - 4 \times D)$$

Secondarily, we also add the following terms to the Moon's Heliocentric Latitude.

$$-0.173^o \times \sin(F - 2 \times D)$$

$$-0.055^o \times \sin(M_m - F - 2 \times D)$$

$$-0.046^o \times \sin(M_m + F - 2 \times D)$$

$$+0.033^{o} \times \sin(F + 2 \times D)$$

$$+0.017^{o} \times \sin(2 * M_m + F)$$

Finally, we also add the following terms to M_r, the Lunar Distance

$$-0.58 \times \cos(M_m - 2 \times D)$$

$$-0.46 \times \cos(2 \times D)$$

Jupiter, Saturn, & Uranus

Before computing the Perturbations for Jupiter, Saturn, and Uranus, we recognize the following shorthands:

M_j = Jupiter's Mean Anomaly

M_s = Saturn's Mean Anomaly

M_u = Uranus' Mean Anomaly

Jupiter

To account for Jupiter's Perturbations, we add the following terms to its Heliocentric Longitude.

$$-0.332^{o} \times \sin(2 \times M_j - 5 \times M_s - 67.6^{o})$$

$$-0.056^{o} \times \sin(2 \times M_j - 2 \times M_s + 21^{o})$$

$$+0.042^{o} \times \sin(3 \times M_j - 5 \times M_s + 21^{o})$$

$$-0.036^{o} \times \sin(M_j - 2 \times M_s)$$

$$+0.022^{o} \times \cos(M_j - M_s)$$

$$+0.023^{o} \times \sin(2 \times M_j - 3 \times M_s + 52^{o})$$

$$-0.016^{o} \times \sin(M_j - 5 \times M_s - 69^{o})$$

Saturn

To account for Saturn's Perturbations, we first add the following terms to its Heliocentric Longitude.

$$+0.812^{o} \times \sin(2 \times M_j - 5 \times M_s - 67.6^{o})$$

$$-0.229^{o} \times \cos(2 \times M_j - 4 \times M_s - 2^{o})$$

$$+0.119^{o} \times \sin(M_j - 2 \times M_s - 3^{o})$$

$+0.046^{\circ} \times \sin(2 \times M_j - 6 \times M_s - 69^{\circ})$

$+0.014^{\circ} \times \sin(M_j - 3 \times M_s + 32^{\circ})$

Secondarily, we also add the following terms to Saturn's Heliocentric Latitude.

$-0.020^{\circ} \times \cos(2 \times M_j - 4 \times M_s - 2^{\circ})$

$+0.018^{\circ} \times \sin(2 \times M_j - 6 \times M_s - 49^{\circ})$

▼ Uranus

To account for Uranus' Perturbations, we add the following terms to its Heliocentric Longitude.

$+0.040^{\circ} \times \sin(M_s - 2 \times M_u + 6^{\circ})$

$+0.035^{\circ} \times \sin(M_s - 3 \times M_u + 33^{\circ})$

$-0.015^{\circ} \times \sin(M_j - M_u + 20^{\circ})$

▼ Pluto

Note that Pluto's Orbital Elements have not yet been determined. For this reason we rely on more detailed but hardcoded equations to provide us with Pluto's Heliocentric Ecliptic Coordinate.

$$
\begin{aligned}
l = \\
& 238.9508 + 0.00400703 \times d \\
& -19.799 \times \sin(P) + 19.848 \times \cos(P) \\
& +0.897 \times \sin(2 \times P) - 4.956 \times \cos(2 \times P) \\
& +0.610 \times \sin(3 \times P) + 1.211 \times \cos(3 \times P) \\
& -0.341 \times \sin(4 \times P) - 0.190 \times \cos(4 \times P) \\
& +0.128 \times \sin(5 \times P) - 0.034 \times \cos(5 \times P) \\
& -0.038 \times \sin(6 \times P) + 0.031 \times \cos(6 \times P) \\
& +0.020 \times \sin(S - P) - 0.010 \times \cos(S - P)
\end{aligned}
$$

$$
\begin{aligned}
b = \\
& -3.9082
\end{aligned}
$$

$$-5.453 \times \sin(P) - 14.975 \times \cos(P)$$
$$+3.527 \times \sin(2 \times P) + 1.673 \times \cos(2 \times P)$$
$$-1.051 \times \sin(3 \times P) + 0.328 \times \cos(3 \times P)$$
$$+0.179 \times \sin(4 \times P) - 0.292 \times \cos(4 \times P)$$
$$+0.019 \times \sin(5 \times P) + 0.100 \times \cos(5 \times P)$$
$$-0.031 \times \sin(6 \times P) - 0.026 \times \cos(6 \times P)$$
$$+0.011 \times \cos(S - P)$$

$$r =$$
$$40.72$$
$$+6.68 \times \sin(P) + 6.90 \times \cos(P)$$
$$-1.18 \times \sin(2 \times P) - 0.03 \times \cos(2 \times P)$$
$$+0.15 \times \sin(3 \times P) - 0.14 \times \cos(3 \times P)$$

Geocentric Placements

One Perturbations have been accounted for in the relevant Orbitals, we can now move

~~Retrograde Motion~~

House System Computations

Cardinal Points of The Chart

If you're familiar with Astrology, you may have seen the Ascendant and Midheaven listed alongside other Planetary Placements. If you're particularly adept, you may already know that they are not Planets, but are in fact Houses in most cases and relevant to interpretation in all cases. One of the beauties of the many House Systems enumerated below is that they all share these Cardinal Points unless otherwise stated. So while the methodologies for obtaining House Cusps can vary greatly, they are at large unified by these two reference points and their diametric counterparts.

The relationship of these points to the Coordinate Systems we have established is explained here.

- ▼ Ascendant / Descendant

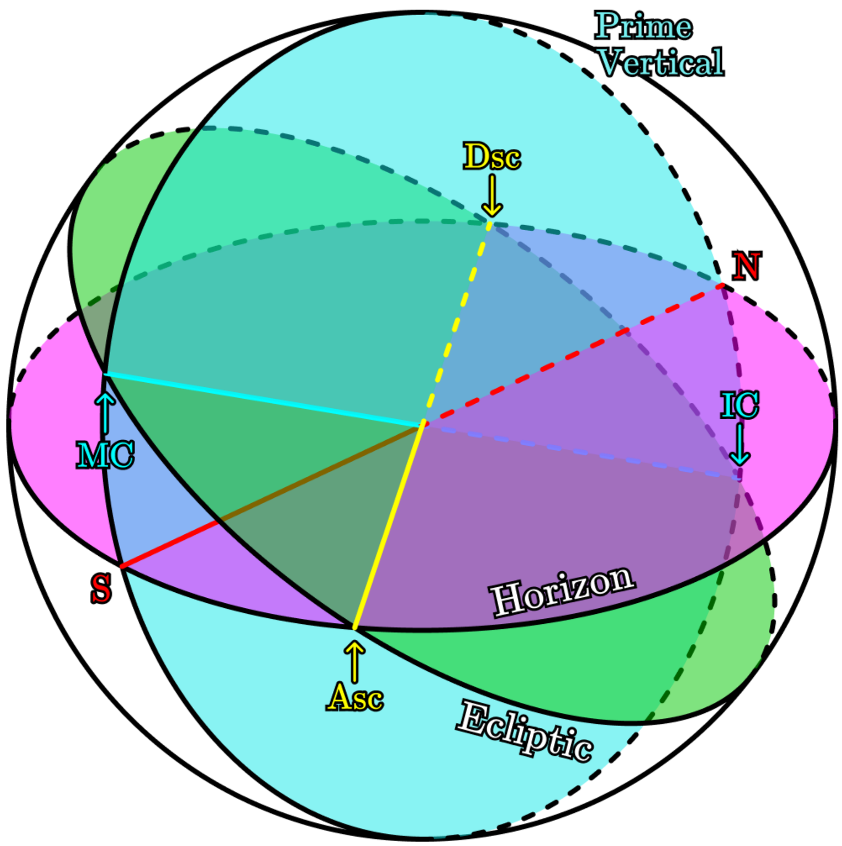
  The Ascendant can be found at the Easternmost intersection of the Horizontal Plane and the Ecliptic Plane. To compute this, we can take the Normal Vector of the Horizontal Plane (The Zenith), converting it first to Equatorial Space and secondly to Geocentric Ecliptic Space. Once there, we can compute its cross product with the Normal Vector of the Geocentric Ecliptic Plane to find the resulting intersection. This intersection defines both the Ascendant and Descendant.

  In many but not all House Systems, the Ascendant is the 1st House Cusp and the Descendant is the 7th House Cusp.

- ▼ Midheaven / Imum Coeli

  The Midheaven can be found at the point where the secondary Great Circle of the Horizontal Coordinate System (The Prime Vertical) intersects with the Ecliptic Plane.

  In many but not all House Systems, the Midheaven is the 10th House Cusp and the Imum Coeli is the 4th House Cusp.



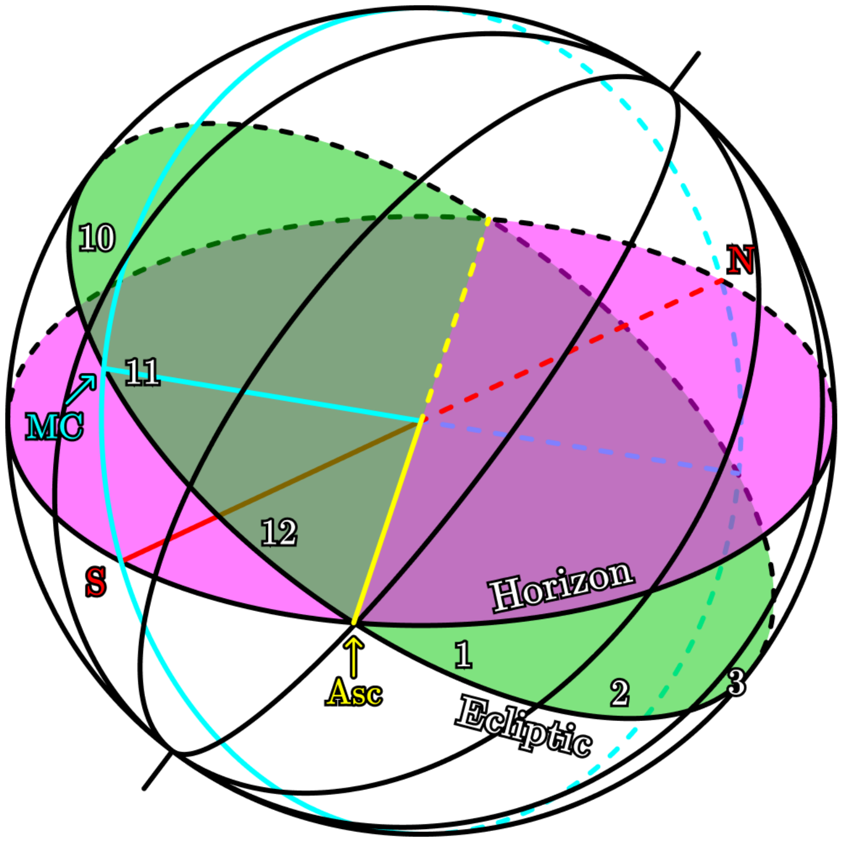
▼ Ecliptic-Based Systems

Ecliptic-Based Systems posit that the means of determining the House Cusps should be contained entirely within the realm of the Ecliptic Plane. Being the primary Great Circle of interest in the Astrological tradition and the home of all Placements via projection, this was the apparent and intuitive choice. As a result, the following systems can be performed directly without the need for Coordinate Conversion.

▼ Equal Sign Houses

Equal Houses are arguably the oldest method of dividing the houses and by far the most mathematically simple. Simply compute the Ascendant and utilize it as a reference point. This marks the first House Cusp, and all subsequent House Cusps are computed by adding equal, 30° angle increments along the Ecliptic Plane.

As a consequence of this,

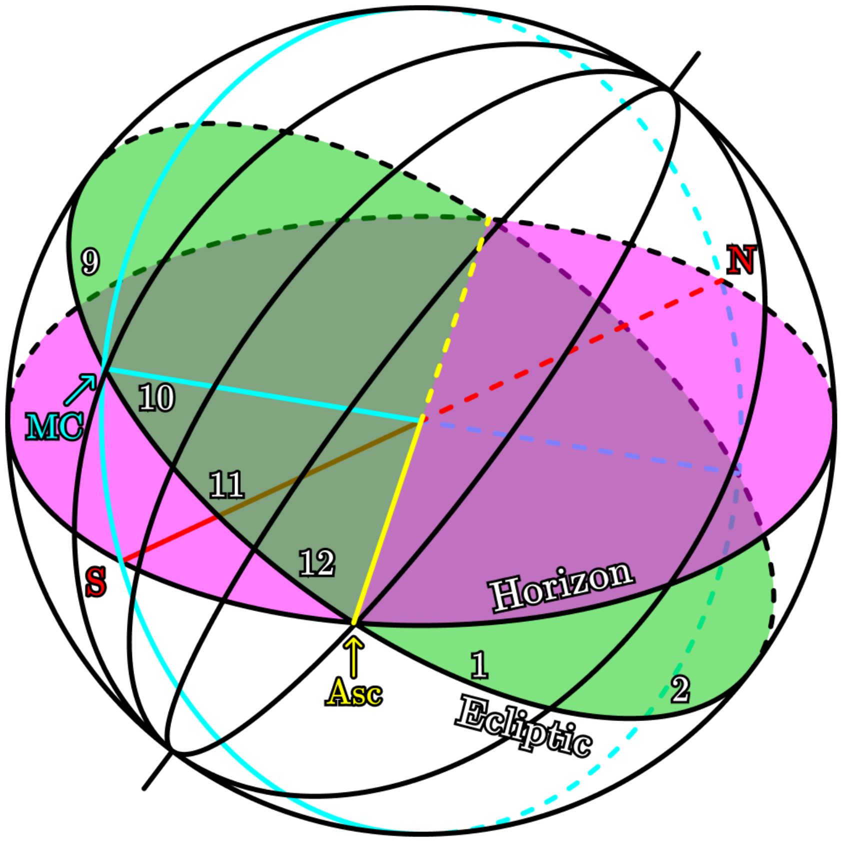


▼ Whole Sign Houses

Whole Sign Houses are very alike Equal Houses in both age and complexity. First, compute the Ascendant. Now ask, which Sign does the Ascendant fall under? The 0^o of this Sign will be the mark of the first House Cusp, and all subsequent House Cusps will be computed in equal, 30^o angle increments along the Ecliptic Plane.

▼ Porphyry Houses

Porphyry Houses serve to add an important revelation in the world of House Division. Unlike in the case of Equal and Whole Houses, Porphyry Houses maintain that the Midheaven should serve as the 10th House Cusp. Now, rather than starting with the Ascendant and adding 30^o increments along the Ecliptic Plane, we start with the Ascendant, Midheaven, Descendant, and Imum Coeli, equally dividing each quadrant of the Ecliptic into three parts.



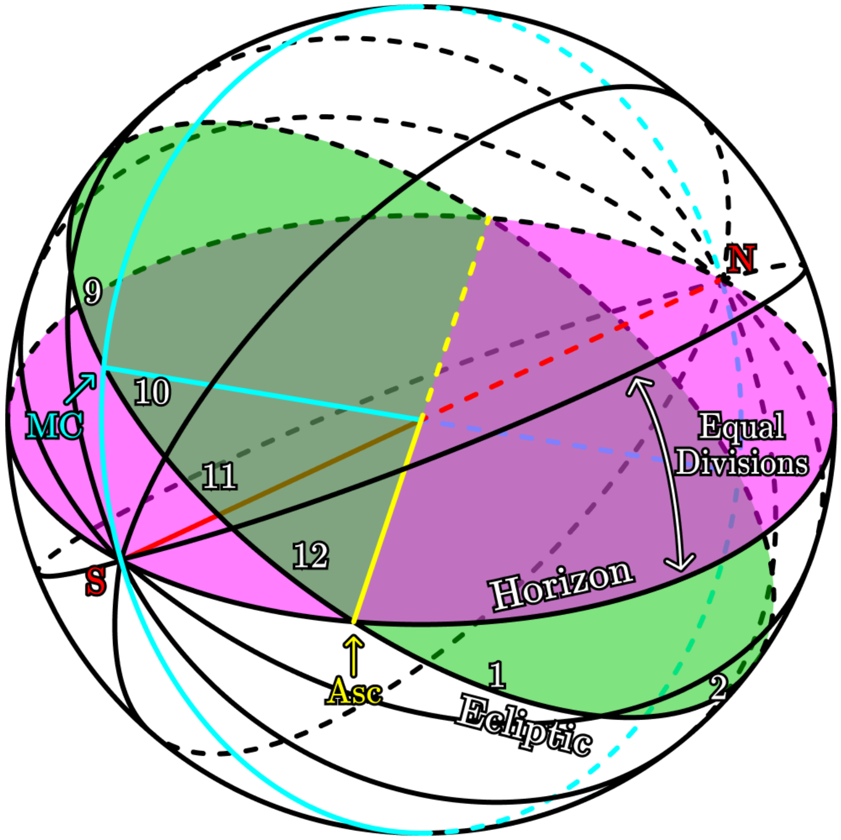
▼ Space-Based Systems

Space-Based Systems depart from the Ecliptic paradigm. Rather than directly dividing the Ecliptic Plane in some predetermined way, we now posit that it is actually the Celestial Sphere that should be equally divided in some meaningful way. From these divisions, relationships to the Ecliptic Plane are subsequently inferred and used to arrive at final House Division longitudes.

▼ Campanus Houses

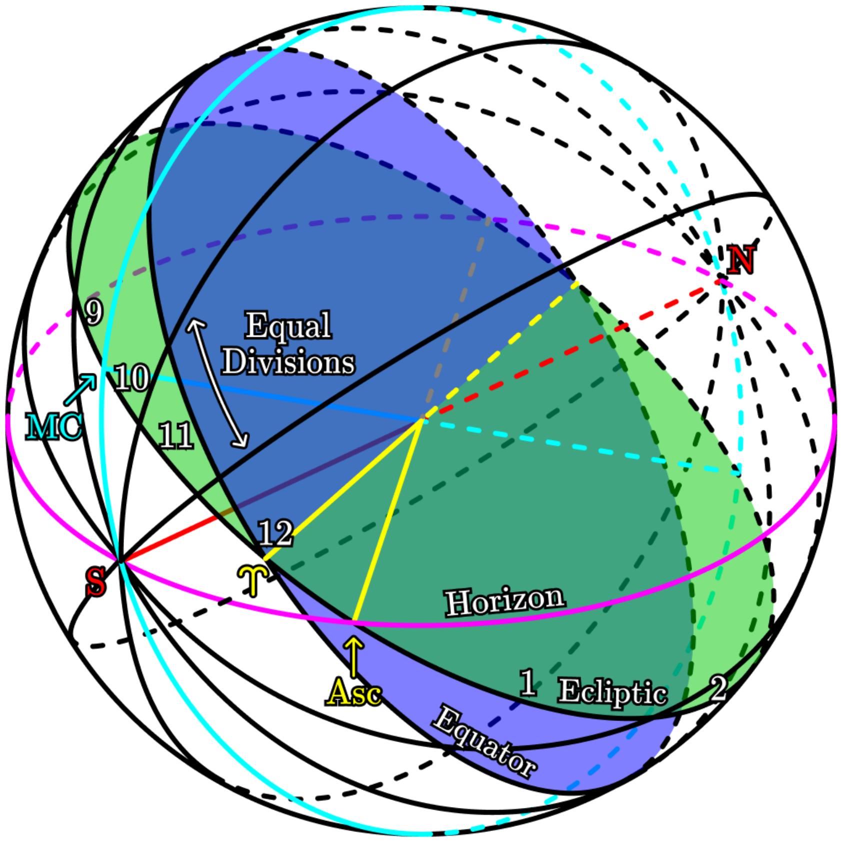
Campanus iterated on the conceptual work done by Meridian and Morinus Houses, maintaining the equal division of the Celestial Sphere. Building on these concepts, he integrated them with Porphyry's notion of congruency between House Cusps and the Ascendant, Midheaven, Descendant, and Imum Coeli.

To construct Campanus Houses, we begin with the Midheaven and create equal divisions perpendicular to the Prime Vertical such that the House Cusps intersect with the North and South points on the Horizon. These lines are then traced through the Ecliptic Plane, each intersection marking a House Cusp.



▼ Regiomontanus Houses

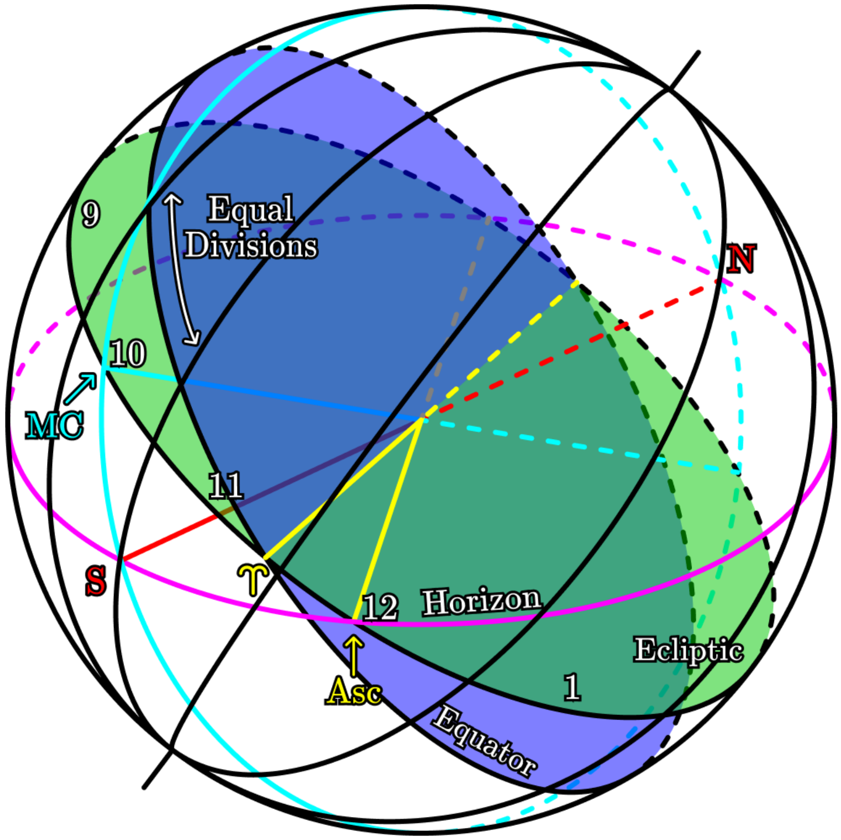
To construct Regiomontanus Houses, we begin by computing the intersection of the Prime Vertical and the Equator. From here, equal increments of 30º are added to the computed Coordinate's Right Ascension. From each of these reference points, Great Circles are drawn through the North and South points of the Horizontal Plane. The intersections of these Great Circles with the Ecliptic Plane are the House Cusps.



▼ Morinus Houses

To construct Morinus Houses, we find the intersection of the Prime Vertical and the Equator. In doing so, we mark this point on the Equator as our 10th House, and subsequently make equal divisions of Right Ascension around the Equator. House Cusps are then determined by tracing lines through these equatorial points as well as the poles of the Ecliptic Plane.

Morinus Houses benefit from the fact that the Equatorial and Ecliptic Planes remain at a relatively fixed angle to one another, not accounting for Precession. Therefore even though the 10th House can begin at an arbitrary location, the fact that the equal divisions are made along the Equatorial Plane prevents the distortion of House Cusps at more extreme latitudes. This is the highlighted feature of the Morinus Houses, though they experience the caveat that the Cardinal Points of a given chart are not congruent with the Houses as Porphyry first posited should be the case.



▼ ~~Time-Based Systems~~

Fundamentally different from the systems we've seen thus far, Time-Based House Systems rely on the computation and prediction of rising, transiting, and setting times.

▼ Alcabitius Houses

▼ Placidus Houses

Further Reading & Resources

1. https://stjarnhimlen.se/comp/tutorial.html
2. https://stjarnhimlen.se/comp/ppcomp.html
3. https://stjarnhimlen.se/comp/riset.html

4. https://en.wikipedia.org/wiki/Astronomical_coordinate_systems
5. http://neoprogrammics.com/sidereal_time_calculator/index.php